AIAA JOURNAL
Vol. 30, No. 10, October 1992

Assessment of Compressibility Corrections to the k-ϵ Model in High-Speed Shear Layers

John R. Viegas* and Morris W. Rubesin†
NASA Ames Research Center, Moffett Field, California 94035

Abstract

TWO compressibility corrections to the standard k-ϵ turbulence model are used with the Navier-Stokes equations to compute the mixing region between two streams of the same gas flowing under a variety of low- and high-speed freestream conditions. The model corrections are assessed by comparing 1) computed spread rates for a series of unconfined flows with data for a range of convective Mach numbers, and 2) velocity profiles for a confined flow.

Contents

Since 1972, it has been known that application of the classical k-ϵ turbulence model in parabolic computations of the flow in single-stream free shear layers shows more spreading than actually occurs.[1] Several modifications to either k-ϵ or full Reynolds stress modeling have recently been developed that purport to be able to correct this modeling deficiency.[2-4] Here, the relative performances of two compressibility corrections applied to the k-ϵ turbulence model, as well as its standard form, are assessed for free-shear layers under a variety of high-speed freestream conditions by comparing the computed flowfields with experimental data.[5-10] Results are also presented for a shear layer developing within the confines of walls and with boundary layers of finite thickness on an upstream splitter plate between the streams.[11]

In this study, a fully implicit Navier-Stokes solver[12] was employed, which allowed departure of conditions from thin-layer or boundary-layer assumptions. The corrected turbulence models, identified as the SEHK and Zeman theories, Refs. 2 and 3, respectively, were developed originally as full second-order closure models. Here, only their modifications to the dissipation rate $\rho\epsilon$ in the standard turbulence kinetic energy equation are employed. These dissipation rates are as follows:

SEHK:

$$\rho\epsilon(1 + M_t^2), \qquad M_t^2 = 2k/(\gamma RT)$$

Zeman:

$$\rho\epsilon[1 + 0.75F(M_{tc})]$$

$$F(M_{tc}) = (1 - \exp\{-[(M_{tc} - 0.1)/0.6]^2\})H(M_{tc} - 0.1)$$

where $M_{tc}^2 = (1 + \gamma)M_t^2/2$, $H(x)$ is the Heaviside step function and M_t and M_{tc} are turbulence Mach numbers (M_{tc} is based on the critical speed).

For the unbounded free-shear layers, two types of flow conditions were considered for the mixing streams. In both cases, the static pressures of the streams were matched. For most of the examples, the total temperatures of the streams were also matched. For others, however, the densities of the mixing streams were matched.

Two alternative correlation techniques for comparing the spread rates were employed. The first represents the spreading rate of a free-shear layer in compressible flow as

$$\frac{d\delta_{pit}}{dx} = G(M_c)(1 - r)(1 + \sqrt{s})/(1 + r\sqrt{s}) \tag{1}$$

where $r = U_2/U_1$, $s = \rho_2/\rho_1$, subscripts 1 and 2 correspond to the high-speed and low-speed streams, respectively. M_c is a convective Mach number defined as

$$M_c = (M_1\sqrt{s} - M_2)/(1 + \sqrt{s}) \tag{2}$$

for streams composed of the same species. The shear layer thickness δ_{pit} at a given station is the distance between points in the free-shear layer where the local impact pressures are 0.05 and 0.95 of the difference in the local freestream impact pressures on either side of the free-shear layer. The value of the function $G(M_c)$ is found from Eq. (1) by comparing spread rates found from experiments or calculations for various values of M_c. The effect of compressibility, distinct from density ratios in the streams, is then expressed as $G(M_c)/G(0)$.

A second way of correlating free-shear layer spread rate data[9] is to employ the vorticity thickness defined as $\delta_\omega = (U_1 - U_2)/(\partial U/\partial y)_{max}$. With this definition, the spread rate is represented by

$$\frac{d\delta_\omega}{dx} = C_\omega(M_c)(1 - r)/(1 + r) \tag{3}$$

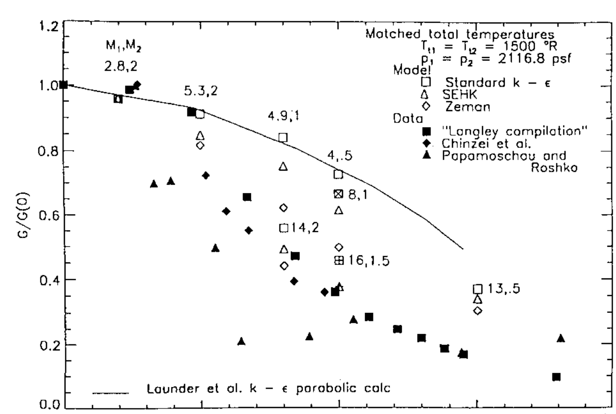

Fig. 1 Effect of compressibility on normalized impact pressure thickness growth rate.

Received June 14, 1991; presented as Paper 91-1783 at the AIAA 22nd Fluid Dynamics, Plasma Dynamics, and Lasers Conference, Honolulu, HI, June 24-26, 1991; synoptic received Aug. 6, 1991; accepted for publication Dec. 7, 1991. Full paper available from AIAA Library, 555 W. 57th St., New York, NY 10019.

*Research Scientist, Experimental Fluid Dynamics Branch, Mail Stop 229-1. Associate Fellow AIAA.

†Senior Scientist, MCAT Institute, Mail Stop 229-1. Fellow AIAA.

The effects of compressibility are represented by $C_\omega(M_c)/C_\omega(0)$, where $C_\omega(M_c)$ is found from experimental measurements or from calculations. Low Mach number and $T_1 = T_2$ computations were used to establish $C_\omega(0) = 0.168$ and $G(0) = 0.079$ with all the models.

Figures 1 and 2 summarize the comparison of experimental and computed spread rates of free-shear layers based on the two definitions of thickness and on different turbulence models. All of the data and computed cases have matched total temperatures. This is consistent with the assumption that the experimental data for single streams flowing over backward-facing steps were obtained with adiabatic wall conditions and recovery factors close to unity. The total temperatures, static pressures, and Mach numbers for the two streams as well as the turbulence model used in the computations are indicated in the figures. The experimental data are identified by solid symbols, and in Fig. 1 the solid line represents the parabolic computations of the standard k-ϵ model by Launder et al.[1] Open symbols filled by a cross or an x differentiate between cases where overlapping occurs.

This consistently defined treatment yields better agreement between the different sets of data than has appeared in many recent papers where all sorts of spread rate definitions are mixed together indistinguishably. In particular, the data of the "Langley compilation" converted to these coordinates agree reasonably well with the behavior of the Papamoschou and Roshko data. Also, the Launder et al. k-ϵ line on Fig. 1 shows

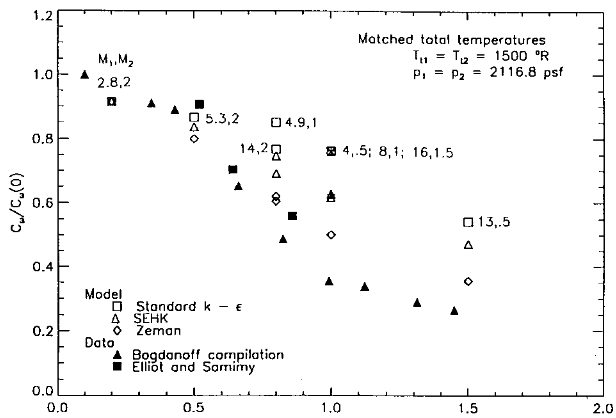

Fig. 2 Effect of compressibility on normalized vorticity thickness growth rate.

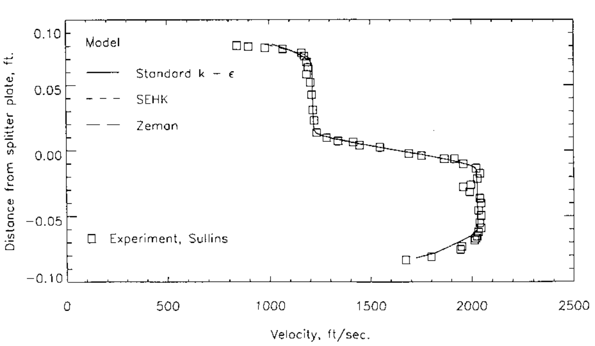

Fig. 3 Comparison of measured and computed streamwise mean velocity profiles of air with $M_1 = 3.06$ and $M_2 = 1.25$. Streamwise distance from splitter plate = 5.94 in.

some reduction in spread rate with M_c that was not apparent in the coordinate system of Ref. 1.

Figures 1 and 2 each show the experimental decrease in the spread rate with increased M_c. The computed results with the standard k-ϵ model do not predict the extent of the reduction. Each tested compressibility correction reduces the spread rate, with the Zeman model giving the largest reduction and yielding results most like the data. However, even this model overpredicts the spread rate by about 30% at $M_c = 1.5$. Agreement exists between the present standard model two-stream and Launder's single-stream calculations for relatively small differences between M_1 and M_2. Reduced spread rates occur for the standard model as the differences between M_1 and M_2 become very large for fixed M_c. This Mach number effect (fixed M_c, but variable M_1 or M_2) contributes to the vertical spread in the data on this figure. In Fig. 2, however, the large Mach number effect in Fig. 1 has very nearly vanished. This suggests that the vorticity thickness correlation function is far superior to the impact pressure thickness correlation for spread rate comparisons. Furthermore, although the Zeman model was the best of the three examined here, it can still stand improvement.

Figure 3 shows comparison of measured and computed velocity profiles of a confined shear layer.[11] This is a flow with an M_c of about 0.5 and for which previous computations with the standard k-ϵ model had agreed quite well with the mean values of the experimental velocities, and there was concern that accounting for compressibility would introduce effects to upset this agreement. It is satisfying that the compressibility corrections did not alter the good results previously obtained with the standard model.

References

[1]Launder, B. E., Morse, A., Rodi, W., and Spalding, D. B., "Prediction of Free Shear Flows—A Comparison of the Performance of Six Turbulence Models," *Free Turbulent Shear Flows,* NASA SP-321, 1972, pp. 361–426.

[2]Sarkar, S., Erlebacher, G., Hussaini, M. Y., and Kriess, H. O., "The Analysis and Modeling of Dilitational Terms in Compressible Turbulence," NASA CR 181959, ICASE Rept. 89-79, Dec. 1989.

[3]Zeman, O., "Dililation Dissipation: The Concept and Application in Modeling Compressible Mixing Layers," *Physics of Fluids,* Vol. 2, No. 2, 1990, pp. 178–188.

[4]Rubesin, M. W., "Extra Compressibility Terms for Favre-Averaged Two-Equation Models of Inhomogeneous Turbulent Flows," NASA CR 177556, June 1990.

[5]Birch, S. F., and Eggers, J. M., "A Critical Review of the Experimental Data for Developed Free Turbulent Shear Layers," *Free Turbulent Shear Flows,* NASA SP-321, 1972, pp. 11–40.

[6]Kline, S. J., Cantwell, B. J., and Lilley, G. M. (eds.), *1980–81 AFOSR-HTTM-Stanford Conference,* Vol. 1, Stanford Univ. Press, Stanford, CA, 1982, p. 368.

[7]Papamoschou, D., and Roshko, A., "The Compressible Turbulent Shear Layer: An Experimental Study," *Journal of Fluid Mechanics,* Vol. 197, 1988, pp. 453–477.

[8]Samimy, M., and Elliot, G. S., "Effects of Compressibility on the Characteristics of Free Shear Layers," *AIAA Journal,* Vol. 28, No. 3, 1990, pp. 439–445.

[9]Bogdanoff, D. W., "Compressibility Effects in Turbulent Shear Layers," *AIAA Journal,* Vol. 21, No. 6, 1983, pp. 926–927.

[10]Chinzei, N., Masuya, G., Komuro, T., Murakami, A., and Kudou, K., "Spreading of Two-Stream Supersonic Turbulent Mixing Layers," *Physics of Fluids,* Vol. 29, No. 5, 1986, pp. 1345–1347.

[11]Sullins, G., "Shear Layer Mixing Tests," Applied Physics Lab., Johns Hopkins Univ., Laurel, MD, April 1989.

[12]MacCormack, R. W., "A Numerical Method for Solving the Equations of Compressible Viscous Flow," *AIAA Journal,* Vol. 20, No. 9, 1982, pp. 1275–1281.